Lifting Based DWT for Lossy Image/Signal Compression

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Abstract. A new wavelet transform algorithm based on a modified lifting scheme is presented. The algorithm uses the wavelet filters derived from a generalized lifting scheme at the decomposition stage. The proposed generalized framework for the lifting scheme permits to obtain easily different wavelet filter coefficients in the case of the (~*N*, *N*) lifting. The restoration stage of the lifting scheme was modified in a standard wavelet transform fashion to obtain good rate/distortion performance. The algorithm performs decomposition/restoration on the first level whereas at the next level classic wavelet filters are used. For this purpose, the restoration high-pass wavelet filter was derived from the generalized lifting scheme The obtained restoration high-pass and low-pass wavelet filters operate as a known classic wavelet filters using data upsampling. The low-pass filter for the restoration stage uses the same coefficients of the lifting decomposition predictor. The proposed transform possesses the perfect restoration of the processed data when the quantification is not used and possesses a good energy compactation. The designed algorithm was tested on different test images and the obtained results are comparable with the JPEG2000 9/7 wavelet filtering. The proposed algorithm performs better on the images having high energy of the low-scale details.

Keywords: image processing, wavelets, lifting scheme, lossy data compression

1 Introduction

In the past decade, the wavelet transform has become a popular, powerful tool for different image and signal processing applications such as noise cancellation, data compression, feature detection, etc. Meanwhile, the aspects of fast wavelet decomposition/reconstruction implementation and high rate data compression now continue to be under consideration.

The first algorithm of the fast discrete wavelet transform (DWT) was proposed by S.G.Mallat [1]. This algorithm is based on the fundamental work of Vetterli [2] on signal/image decomposition by 1-D quadrature-mirror filters (QMF), and orthogonal wavelet functions. The next breakthrough concerning a fast DWT algorithm implementation was made by W.Sweldens [3] who first proposed the lifting scheme.

The lifting scheme is known to perform well in lossless data compression. Unfortunately, it has worse rate/distortion characteristics than the classic wavelets and for this reason is not employed for lossy data compression. In this paper, we try to overcome such lifting scheme inefficiency.

We present a DWT algorithm, which is based on the proposed generalized lifting scheme framework. According to this framework, different wavelet filters can be easily obtained. With the proposed method, varying some coefficients, one can obtain the wavelet filter of a different order and frequency response to adjust the wavelet decomposition properties. This way, the decompositions can be optimized to achieve a minimum of the entropy in the wavelet domain.

2 Lifting Scheme Generalization

The lifting scheme is widely used in the wavelet based image analysis. Its main advantages are: the reduced number of calculations; less memory requirements; the possibility of the operation with integer numbers. The lifting scheme consists of the following basic operations: splitting, prediction and update.

Splitting is sometimes referred to as the lazy wavelet. This operation splits the original signal $\{x\}$ into odd and even samples:

$$s_i = x_{2i}, \quad d_i = x_{2i+1}. \tag{1}$$

Prediction, or the dual lifting. This operation at the level k calculates the wavelet coefficients, or the details $\{d^{(k)}\}$ as the error in predicting $\{d^{(k-1)}\}$ from $\{s^{(k-1)}\}$:

$$d_i^{(k)} = d_i^{(k-1)} + \sum_{j=-\tilde{N}/2}^{\tilde{N}/2} p_j s_{i+j}^{(k-1)}, \tag{2}$$

where $\{p\}$ are coefficients of the wavelet-based high-pass FIR filter and $\tilde{N}$ is the prediction filter order.

Update, or the primal lifting. This operation combines $\{s^{(k-1)}\}$ and $\{d^{(k)}\}$, and consists of low-pass FIR filtering to obtain a coarse approximation of the original signal $\{x\}$:

$$s_i^{(k)} = s_i^{(k-1)} + \sum_{j=-N/2}^{N/2} u_j d_{i+j}^{(k-1)}, \tag{3}$$

where $\{u\}$ are coefficients of the wavelet-based low-pass FIR filter and N is the prediction filter order.

Sometimes, the normalization factors can be applied to the wavelet details and approximations to produce the transformed data values more similar to the classic DWT algorithm.

The inverse transform is straightforward: first, the signs of FIR filter coefficients $\{u\}$ and $\{p\}$ are switched; the inverse update followed by inverse prediction is calculated. Finally, the odd and even data samples are merged.

A fresh look at the lifting scheme first was done in [4], where the FIR filters that participate in the prediction and update operation are described in the domain of Z-transform. According to this approach, the transfer function of the prediction FIR filter can be formulated as follows [5]

$$H_p(z) = 1 + p_0\left(z + z^{-1}\right) + p_1\left(z^3 + z^{-3}\right) + \dots + p_{\frac{\tilde{N}}{2}-1}\left(z^{\tilde{N}-1} + z^{-\tilde{N}+1}\right) \tag{4}$$

The $H_p(z)$ must has zero at $\omega = 0$, i.e., at $z = 1$. It can be easily found [4] that this condition is satisfied when

$$\sum_{l=0}^{\frac{\tilde{N}}{2}-1} p_l = -\frac{1}{2}. \tag{5}$$

When the condition (5) is satisfied, $H_p(-1) = 2$ and $H_p(0) = 1$ that means the prediction filter has gain 2 at $\omega = \pi$ and unit gain at $\omega = \frac{\pi}{2}$. Thus, if one want to obtain the normalized detail coefficients, the normalization factor must be equal to $\frac{1}{2}$.

Following this approach, the transfer function for update filter can be obtained. We prefer to formulate this transfer function in the terms of $H_p(z)$ [5]:

$$H_u(z) = 1 + H_p(z)\left\{u_0\left[(z) + \left(z^{-1}\right)\right] + u_1\left[\left(z^3\right) + \left(z^{-3}\right)\right] + \dots + u_{\frac{N}{2}-1}\left[\left(z^{N-1}\right) + \left(z^{-N+1}\right)\right]\right\}. \tag{6}$$

Similarly, $H_u(z)$ must has zero at $\omega = \pi$, i.e., at $z = -1$. It can be easily found [4] that this condition is satisfied when

$$\sum_{l=0}^{\frac{N}{2}-1} u_l = \frac{1}{4}. \tag{7}$$

When the condition (7) is satisfied, $H_u(0) = 1$ and that means the prediction filter has gain 1 at $\omega = 0$.

An elegant conversion of the formulas (5), (7) in the case of (4,4) lifting scheme was proposed in [4] to reduce the degree of freedom in the predictor and update coefficients. With some modifications, the formulas for the wavelet filters coefficients are as follows:

$$p_0 = -\frac{128 + a}{256}, \quad p_1 = \frac{a}{256}, \tag{8}$$

$$u_0 = \frac{64+b}{256}, \qquad u_1 = -\frac{b}{256}, \tag{9}$$

where a and b are the parameters that control the DWT properties. Also, in [4] it was found the correspondence between these control parameters and the conventional (non-lifted) wavelet filters.

The behavior of the lifting scheme with the respect of a and b values can be evaluated easily in the frequency domain using transfer functions (4), (6) with $z = e^{-j\omega}$. Figures 1 and 2 represent the magnitude of the frequency responses of high-pass and low-pass (2,2) and (4,4) filers with $a = 0$, $b = 0$ and $a = 16$, $b = 16$, respectively. These figures were obtained with MATHCAD200i software.

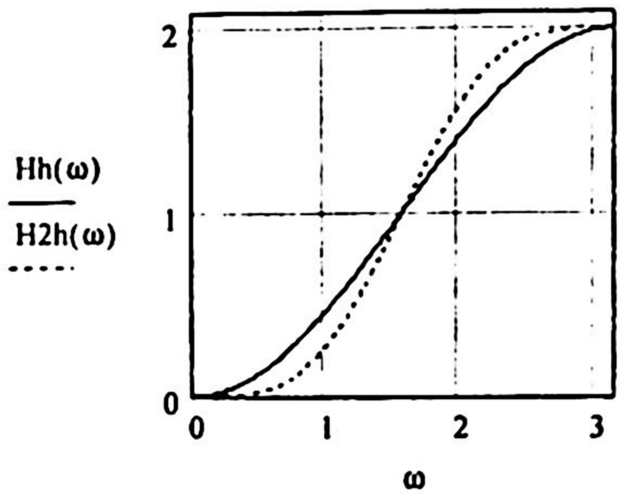

Fig. 1 Frequency response of lifting predictor with $\tilde{N} = 2$ and $a = 0$ ($H2h(\omega)$, dotted line) and $\tilde{N} = 4$ and $a = 16$ ($Hh(\omega)$, solid line)

By simulations with MATHCAD2000i software, we found that better frequency response of (4,4) lifting filter can be obtained with the different a and b parameters. In particular, when $a = 20$, the transition band of the high-pass filter is the narrowest. However, increasing more the coefficient a value leads to the appearance of the false side lobe in the low-frequency band.

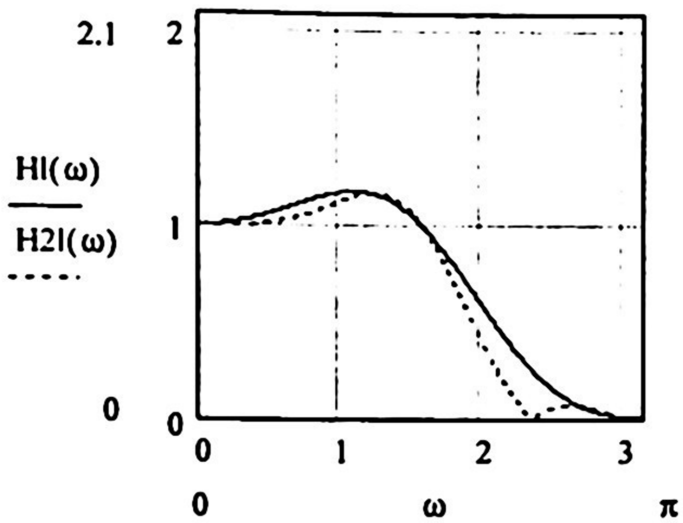

Fig. 2 Frequency response of lifting update filter with $\tilde{N}=2, N=2$ and $a=0, b=0$ ($H2l(\omega)$, dotted line) and $\tilde{N}=4, N=4$ and $a=16, b=16$ ($Hl(\omega)$, solid line)

The behavior of update filter is more complicated. The frequency response of this low-pass filter depends not only on b parameter value, but inherently on the a value. This fact is explained by the nature of the lifting scheme. In practice, the influence of the coefficient a value (which is relatively small) can be avoided choosing this value to produce the predictor with the desired frequency response and then, choosing the appropriate value of the coefficient b. For example, if we choose $a=20$, the narrowest transition band in the update filter is obtained with $b=9$ meanwhile with the greater b a side lobe appears in the high frequency band.

Using the generalization of the lifting scheme (4)- (7), we found by simulations that the coefficients of the lifting filters of an arbitrary order higher than 4 can be found according to the recursive formulas [5]:

$$p_0=-\frac{128+a}{256}, p_1=-\frac{a}{256}, p_2=-\frac{p_1}{c}, p_1=p_1-p_2,\ldots,p_{\tilde{N}}=-\frac{p_{\tilde{N}-1}}{c}, p_{\tilde{N}-1}=p_{\tilde{N}-1}-p_{\tilde{N}} \quad (10)$$

$$u_0=\frac{64+b}{256}, \quad u_1=\frac{-b}{256}, \quad u_2=\frac{u_1}{d}, u_1=u_1-u_2, \ldots, u_N=\frac{u_{N-1}}{d}, u_{N-1}=u_{N-1}-u_{\tilde{N}} \quad (11)$$

where the parameters c, d controls the filter characteristics. This way, the wavelet filters of an arbitrary order can be derived. The formulas (10), (11) are the extension of the equations (8), (9) and use the same parameters a, b to control the characteristics of the lifting filters. The parameters a, b control the width of the transition bands and the new control parameters c, d control the smoothness of the pass and stop bands to prevent the appearance of the lateral lobes: with greater values of a, b the values of c, d tend to be greater. The behavior of the coefficient d is of a rapid increasing, and in practice it has large value sufficiently high to say that the influence of the terms in $H_u(z)$ of the order higher of 3 may be neglected. Thus, in practice, on can use lifting update filter of the order $N <= 6$ without a significant widening of the update filter transition band: the width of this transition band, mainly, is determined by the lifting predictor frequency characteristics. Fig. 3 shows the frequency responses of (10,4) lifting wavelet filters with $a = 28, b = 8, c = 3, d = 3$.

3 Design of Lifting Scheme Based Algorithm for Lossy Data Compression

Generally, the performance of lifting scheme in data compression is worse than of the classic DWT based on subband decomposition, especially in case of lossy compression. This fact can be explained by the following properties of the lifting wavelets.

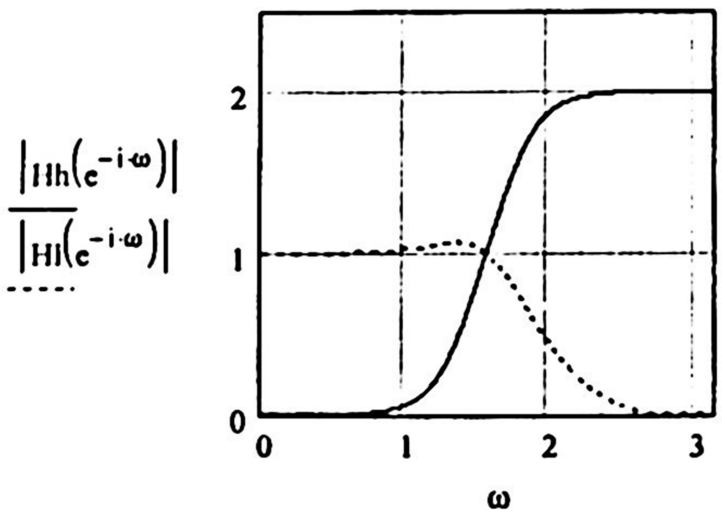

Fig. 3 Frequency responses of (10, 4) lifting filters with $a = 28, b = 8, c = 3, d = 3$

First, the pass band gain of the classic DWT filters frequently is $\sqrt{2}$ meanwhile the lifting filters have gain 1 and 2. The normalization of the gain values of lifting decompositions to $\sqrt{2}$ permits the use of the same quantization technique and facilitates the comparison of the data reconstructed by the different algorithms.

Second, the center of the transition band of high-pass lifting predictor filters is always at $\frac{\pi}{2}$ and the center of the transition band of the low-pass filter is far from the $\frac{\pi}{2}$ tending to the high frequencies. Besides, the transition centers of the classic wavelet filters are optimized to give the aliasing both in high-pass and low-pass filters resulting in better rate/distortion performance for the majority of the natural images. From the other hand, the frequency response of the high-pass lifting filters produces lower values of the wavelet coefficients and, in some situations, the rate/distortion performance of the lifting scheme can be better when the filter characteristics are fit to the spectral characteristics of the image. Such situations are common in case of the images with high energy of the low-scale details. For this reason, the lifting filters usually perform better in the rate/distortion sense at the first level of the wavelet decomposition (when the coefficients a, b, c, d are adjusted properly) meanwhile the classic wavelet filters perform better at higher levels. Another way to solve the problem of the fixed at $\frac{\pi}{2}$ high-pass filter transition band is introduce the even powers of z in Eqs. (4), (6), but such a technique leads to a classic-like wavelet filter design.

The problem of the gain normalization is easy to resolve by introducing the normalization factor in the lifting filters:

$$H_p(z) = \frac{1}{\sqrt{2}} + p_0\left(z + z^{-1}\right) + p_1\left(z^3 + z^{-3}\right) + \ldots + p_{\frac{\tilde{N}}{2}-1}\left(z^{\tilde{N}-1} + z^{-\tilde{N}+1}\right) \qquad (12)$$

$$H_u(z) = \sqrt{2} + H_p(z)\left\{u_0\left[(z) + \left(z^{-1}\right)\right] + \ldots + u_{\frac{N}{2}-1}\left[\left(z^{N-1}\right) + \left(z^{-N+1}\right)\right]\right\} \qquad (13)$$

$$p_0 = -\frac{128+a}{256\sqrt{2}}, \quad p_1 = -\frac{a}{256\sqrt{2}}, \quad p_2 = -\frac{p_1}{c}, p_1 = p_1 - p_2, \ldots,$$

$$p_{\tilde{N}} = -\frac{p_{\tilde{N}-1}}{c}, p_{\tilde{N}-1} = p_{\tilde{N}-1} - p_{\tilde{N}} \qquad (14)$$

$$u_0 = \frac{64+b}{128}, \quad u_1 = \frac{-b}{128}, \; u_2 = \frac{u_1}{d}, u_1 = u_1 - u_2, \ldots u_N = \frac{u_{N-1}}{d}, u_{N-1} = u_{N-1} - u_{\tilde{N}} \qquad (15)$$

The second problem is more complicated. To solve it we propose to employ a hybrid algorithm that uses classic DWT filters at high levels of the decomposition, and the same subband coding technique is used at the first level but with the FIR filters derived from the lifting scheme. To this end, the classic-like wavelet filters need to be found. One can use the inverted predictor (12), (14) for low-pass restoration filtering. At the same time, the high-pass filter for the reconstruction can be derived multiplying directly the polynomials (12), (13). For example, in case of (10,4) lifting the transfer functions of a pair of the reconstruction wavelet filters has the following representation:

$$H_{low-pass}(z) = \frac{1}{\sqrt{2}} + p_0(z+z^{-1}) + .. + p_4(z^9+z^{-9}) \tag{16}$$

$$\begin{aligned} H_{high-pass}(z) &= \sqrt{2} + H_{low-pass}(z)\left[u_0\left(z+z^{-1}\right) + u_1\left(z^3+z^{-3}\right)\right] = \sqrt{2} + 2(p_0u_0 + p_1u_1) \\ &+ u_0\sqrt{2}\left(z+z^{-1}\right) + (p_0u_0 + p_1u_0 + p_0u_1 + p_2u_1)\left(z^2+z^{-2}\right) + u_1\sqrt{2}\left(z^3+z^{-3}\right) \\ &+ (p_1u_0 + p_0u_1 + p_2u_0 + p_3u_1)\left(z^4+z^{-4}\right) + (p_2u_0 + p_1u_1 + p_3u_0 + p_4u_1)\left(z^6+z^{-6}\right) \\ &+ (p_3u_0 + p_2u_1 + p_4u_0)\left(z^8+z^{-8}\right) + (p_3u_1 + p_4u_0)\left(z^{10}+z^{-10}\right) + p_4u_1\left(z^{12}+z^{-12}\right) \end{aligned} \tag{17}$$

The correspondent frequency responses of the filters (16), (17) with $a = 30, b = 8, c = 3, d = 3$ are shown in Fig. 4.

4 Experimental Results

The described in the previous section algorithm were tested on a 128x128 artificial image and a set of 512x512 images ("Lena", "Baboon", "Barbara", "Boat", "Peppers") shown in Fig. 5.

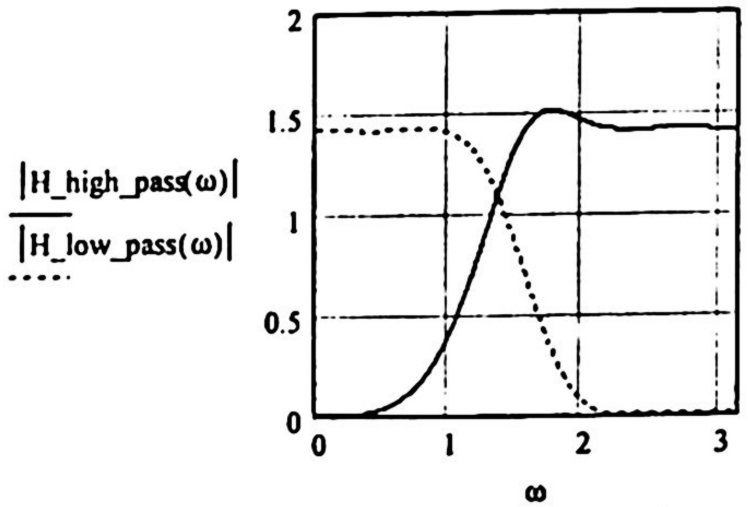

Fig. 4 Frequency responses of the derived from lifting scheme restoration wavelet filters (16),(17)

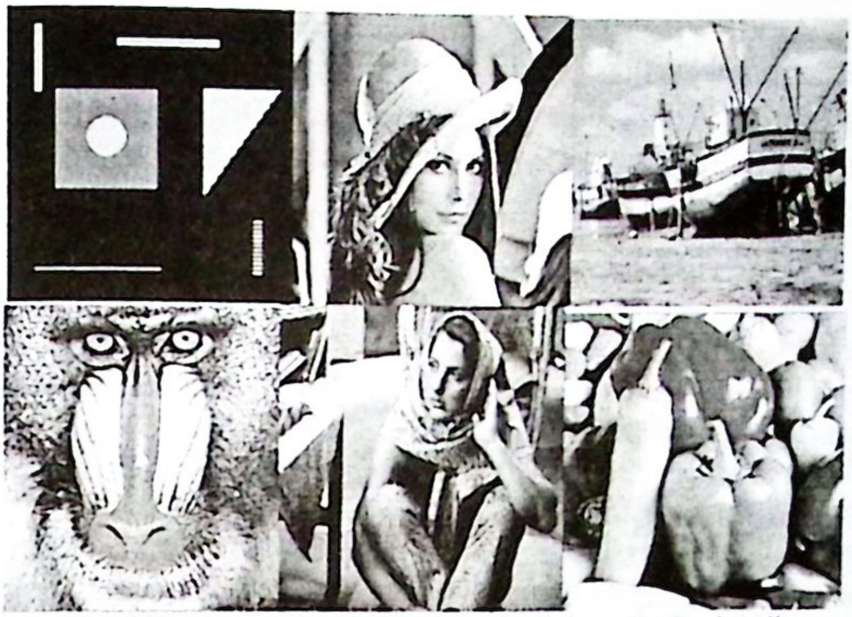

Fig. 5 Test images: artificial, "Lena", "Baboon", "Barbara"

Fig.6 presents the simulation results for the compression of the test images decomposed on <u>one level</u> by the JPEG2000 9/7 wavelet filters and the proposed algorithm. From the analysis of the curves presented in this Figure, it follows that the proposed lifting algorithm performs better than 9/7 wavelets for all test images, especially at high compression rates.

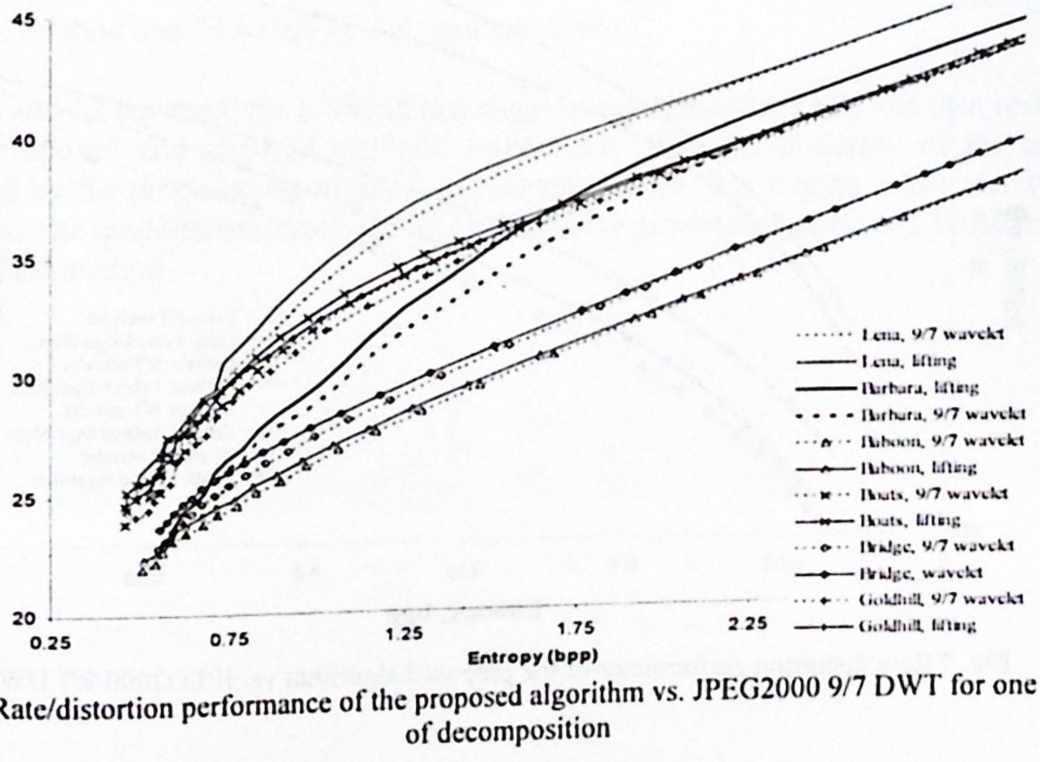

Fig. 6 Rate/distortion performance of the proposed algorithm vs. JPEG2000 9/7 DWT for one level of decomposition

Fig. 7 and 8 show the resulted rate/distortion dependencies obtained with the proposed hybrid algorithm and with JPEG2000 9/7 DWT. In simulations for data compression, the parameters a, b, c, d were varied to obtain the better PSNR y lower entropy per pixel for each test image. The best results were obtained with $a = 20 + 30$, $b = 8 + 12$, $c = 2 + 7$ and $d = 2 + 7$. The order $\tilde{N}$ also were varied and in all cases the best results were obtained for $\tilde{N} = 10$. The better value of order N in all cases was $N = 6$.

It follows from the results presented in the Fig. 7, 8 that the designed algorithm possesses good energy compaction ability comparing to the JPEG2000 9/7 DWT performance. For the test images "Lena" and "Mandrill" the proposed algorithm performs slightly worse. For the artificial test image the proposed algorithm performs significantly better. For the test image "Barbara" the developed algorithm performs slightly better in energy compaction than the JPEG2000 9/7 DWT.

The obtained simulation results show that the energy compaction characteristics of the proposed algorithms depend on the predictor and update filter orders. Generally, for better data compression the higher orders are required. However, in the simulations, the predictor filter order was limited to $\tilde{N} = 10$ and the update filter order was limited to $N = 3$ due to the necessity of analytic calculation of the high-pass restoration filter transfer function for different $\tilde{N}$ and N. These limitations were done because in this paper we only wished to demonstrate that the proposed method is competitive with the standard one.

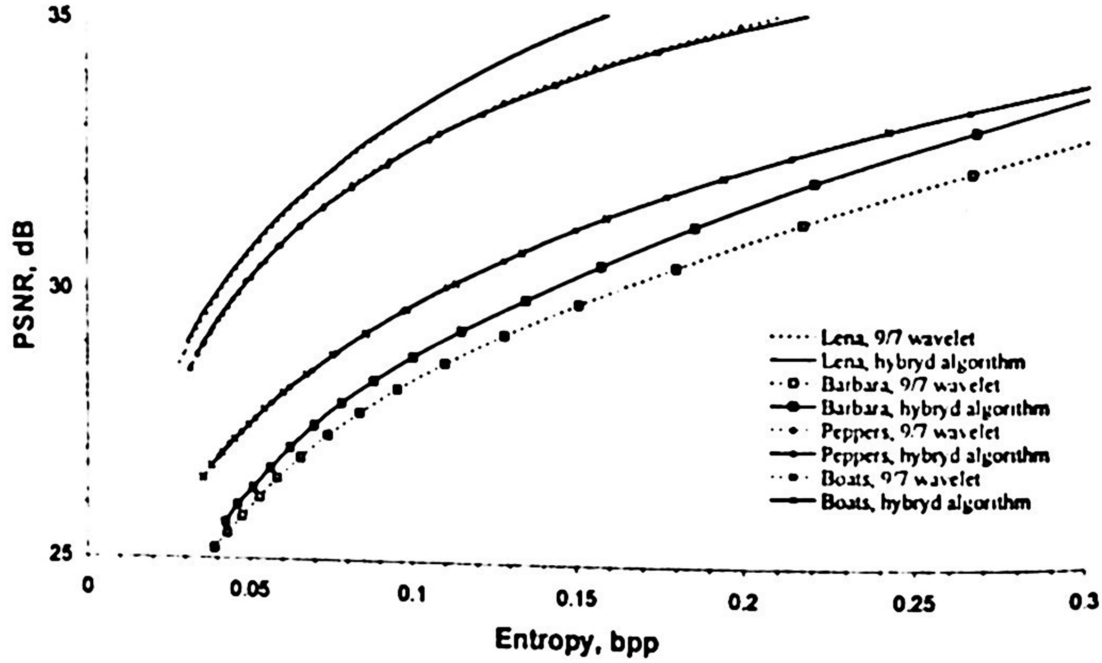

Fig. 7 Rate/distortion performance of the proposed algorithm vs. JPEG2000 9/7 DWT

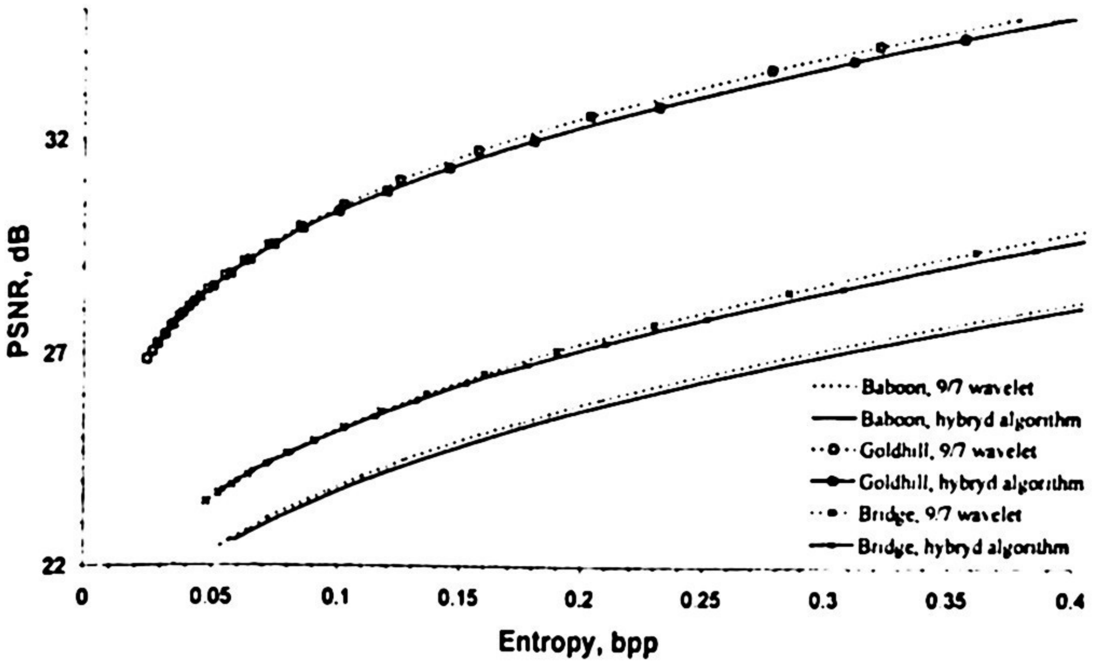

Fig. 8 Rate/distortion performance of the proposed algorithm vs. JPEG2000 9/7 DWT

Fig. 9 and 10 represent the test image "Barbara" compressed to 0.15 bpp and then restored by the proposed and standard methods, respectively. The visual quality of the both images is almost the same but the proposed algorithm produces less ringing in the vicinity of the borders of the objects. The quantitative results are 30.06 dB for the proposed method and 24.61 dB for the standard method.

Fig. 12 and 13 represent the artificial test image compressed to 0.3 bpp and then restored by the proposed and standard methods, respectively. The visual quality of the image processed by the proposed algorithm is significantly better: less ringing in the vicinity of the edges. The quantitative results are 37.15 dB for the proposed method and 35.88 dB for the standard method.

Fig. 8 Test image "Barbara" compressed to 0.15 bpp (normalized entropy value) and restored using the proposed modified lifting algorithm with $a = 28$, $b = 15$, $c = 3$, $d = 2$, $\tilde{N} = 10$, $N = 6$; PSNR=30.06 dB.

Fig. 9 Test image "Barbara" compressed to 0.15 bpp (normalized entropy value) and restored using JPEG2000 9/7 DWT, PSNR=24.61dB.

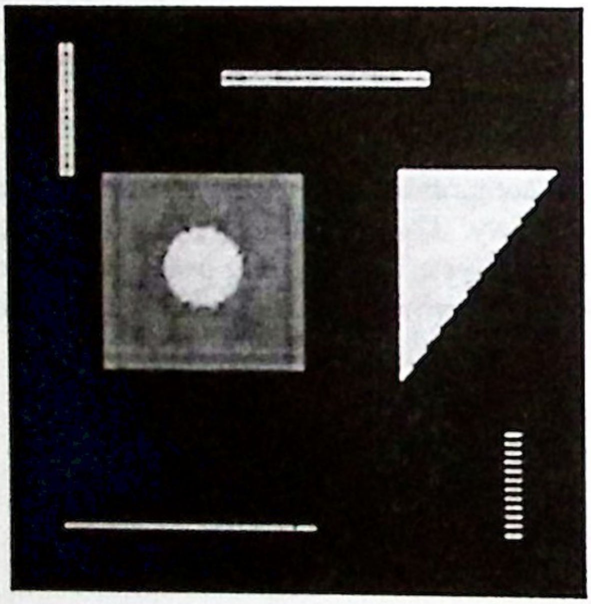

Fig. 10 Artificial test image compressed to 0.3 bpp (normalized entropy value) and restored using the proposed modified lifting algorithm with $a = 4$, $b = 2$, $c = 7$, $d = 6$, $\tilde{N} = 10$, $N = 6$; PSNR=37.15 dB.

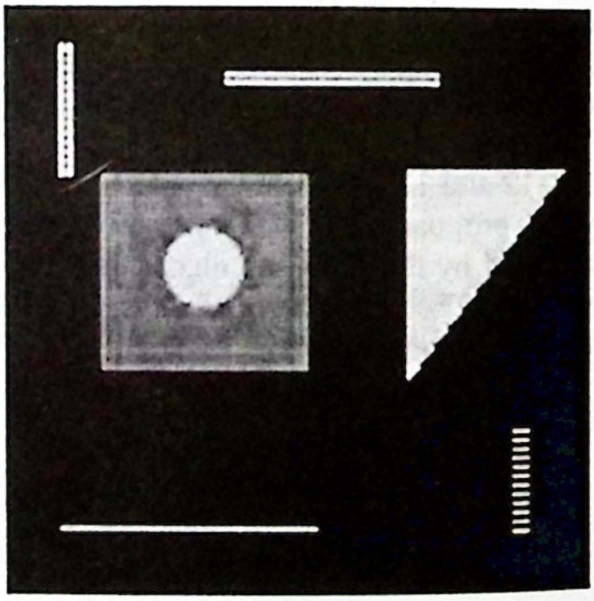

Fig. 11 Artificial test image compressed to 0.3 bpp (normalized entropy value) and restored using JPEG2000 9/7 DWT, PSNR=35.88dB.

5 Conclusion

A novel algorithm of DWT based on the developed generalized lifting scheme is presented. The energy compaction properties of the presented technique are better or comparable in comparison to the standard JPEG2000 wavelet filters. The rate/distortion performance of the proposed algorithm can be enhanced adjusting a set of parameters that gives the possibility to maximize the compression according to the statistical properties of the processed data.

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